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Mathematics News Letter

SUBSCRIPTION $1.00 Per Year

Published under auspices of the Louisiana-Mississippi Section of the Mathematical Association of America and the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics.

To mathematics in general, to the following causes in particular is this journal dedicated: (1) the common problems of grade, high school and college mathematics teaching, (2) the disciplines of mathematics, (3) the promotion of M. A. of A. and N. C. of T. of M. projects.

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VOL. 4 BATON ROUGE, LA., APRIL, 1930 NO. 6

ANOTHER GENEROUS AID

The sympathetic interest of the national body of M. A. of A. in the local struggles and activities of its various Sections has again been evidenced. Due to the initiative of Professor H. E. Slaught, who is ever vigilant in such matters, our News Letter deficit of $43.15, as reported in the recent issue, was brought to the attention of the M. A. of A. executives, with the result that we have recently received from Secretary Cairns a check covering this amount.

All honor to the leaders of this great organization!

An accompanying note from Professor Slaught expressed the strong hope that such a gift from the mother organization might cause a vigorous rally of helpers to the News Letter, particularly from the ranks of mathematical workers in Louisiana and Mississippi.

We must not deceive ourselves. A vast amount of effort is yet due in order to put out and pay for the next two, namely, the May and June, issues.

New subscriptions continue to trickle in, and steadily the list of subscribers is mounting, but unless the subscriptions already booked are, with equal steadiness, promptly renewed on expiration, printing bills must be met by money from other sources.

May this generous act of the National body inspire other subsidies!

—*S.T.S.*

PROFESSOR STONE'S EXPERIMENT

The investigations of Professor Jno. C. Stone to determine the various types of difficulty in algebra that are experienced by high school students will excite the keenest interest on the part of college and secondary teachers of mathematics. His paper, "Finding a Pupil's Difficulties in Algebra," as published in the last issue of the News Letter disclosed a high degree of pedagogical insight. Especially to be remembered is his initial statement, "It is the errors that a pupil makes that gives the teacher a chance to analyze and diagnose his difficulties." Undoubtedly, readers of the Letter would value highly Mr. Stone's analysis of the errors made by the 330 high school graduates in their attempts to answer correctly within 60 minutes the 25 questions submitted to them. In the meanwhile in lieu of a Stone analysis there will be forthcoming from many others various and sundry interpretations of the results. Thus some editorial comment may not be amiss.

(a) Three hundred thirty students, called upon to answer within 60 minutes as many of the 25 questions as they could, being upper half,—some of them honor—students, naturally fell to working as rapidly as possible with the ambition to answer all of them. Thus many errors were made due only to hurry.

(b) Unquestionably a working conception of "general number" on the part even of a fairly bright young mind is difficult at first. Under the best conditions, for the average 16-year old intelligence it is a far cry from $3(2+5)=21$ to $m(n+p)=mn+mp$. Wider yet for him is the gap between "$84/7=12$" and "the quotient of c by b is c/b". Put in other terms, the faculty which generalizes is but scantily developed in the average mind of the

mid-teens—an age better adapted to such questions as, "In what way can 8 young people arrange themselves within a six-passenger car?" than to such an unmotivated query as, "$(a/b+c/d)$ is equal to what?"

(c) In this comment on the Stone experiment we go no further for the present than to suggest that as long as the said young mind is motivated by interests which are predominantly social, so long must the serious problem of the mathematics teacher be: By what method or methods, can I make the beginnings of algebra for that young mind such a part of his social and pleasure-seeking programs that even factoring or the solution of equations or the simplification of forms shall be made to have for him some human interest or even furnish for him a sort of thrill? Of course the question is an old one—far more easily proposed than answered. Without presuming to answer it, never-the-less, we do raise an associated question: The teaching of speling has been effectivly aided by the spelling bee—a socialized method. In smaller degree have the teachers of reading, dramatics, language, called into service the social instincts of the student. Why may not many lesson programs in elementary algebra be invested with a strong social feature by having two of the brightest members of the class "choose sides" for a contest in "factoring" or in "equation solving", or in "formula simplification"? This is but one suggested example of how the fundamental principle, namely, the socialization of an elementary mathematics lesson, may be employed. Other ways of using it may be thought out and tried—doubtless are being tried already—though we doubt that many schools are doing so.

—*S.T.S.*

THE NEWS LETTER A SURVEY MEDIUM

Informing the Mathematics teachers of Mississippi and Louisiana of the significant development in various lines of teaching technique in our own schools, colleges, and universities should be recognized as a function of the News Letter second only to disseminating the findings of our two national mathematical associations. The more thoroughly this field is covered the more valuable will this service be to the recipients of this paper.

Lack of funds prevents a survey of the teaching of mathematics and the only available plan is to secure the hearty cooperation of every teacher of mathematics to further the interest of the News Letter.

Readers can render a genuine service to the News Letter by bringing to the attention of the editors developments in methodology or other fields which have either local signifiance or are susceptible of wider application, or by sending the names of possible contributors, or contributions of their own. If the future of the News Letter is to be assured, work must be started now and continued during the summer with concerted effort. Certainly we should cooperate to increase our subscriptions.

—D.M.F.

A DISCUSSION

By W. PAUL WEBBER
Louisiana State University

At the Cleveland meeting of the Mississippi-Louisiana Section of the Mathematical Association of America, the following question was brought before a round table. "At what stage in teaching first year mathematics should the idea of a general variable be introduced in trigonometry?" Like most round tables this one did not settle the question finally. It occurred to me that some of the readers of the News Letter might wish to take part in the discussion. It is the purpose of this paper to draw out such discussion. What I say, therefore, is to be taken as argument only and not in any sense my final answer, if I have one.

Is it desirable to radically alter the basis of introducing elementary trigonometry, which, in most cases, goes little, if any, beyond numerical trigonometry? The time allotted by administrators, most of whom are not teachers of mathematics, permits little more. Is there any inherent virtue, for beginners, in a geometric introduction to the subject? In the allotted time, can more trigonometry be presented by the traditional method than by a more analytical introduction? Or, should the subject be introduced geometrically and the more general view given

during the course? At what stage is it best to make this change of view point?

For the sake of argument I am going to take the ground that elementary trigonometry is necessarily an incomplete study of the subject for the reasons named above; that for the vast majority of students its use remains largely geometrical after school; and that any attempt to blend both views under present teaching conditions would confuse rather than clarify the problem.

It may be said that the student comes to college with some geometrical knowledge. Trigonometry originated, historically, in geometric situations. Most of the objectives of the customary short course are geometric in character. The geometric treatment is fairly satisfactory for the purposes stated. The analytical view point is covered, to some extent at least, in elementary calculus by those who go beyond the first year in mathematics. There are interesting generalizations of geometrical matter in trigonometry, such as the cosine law of triangles. This law in a single statement includes three important geometric theorems. One of these, the Pythagorean Theorem is classic. Such geometric generalizations are useful and interesting as such. Some time is indeed for such generalizations of the secondary course. Could this be done as well if time were taken for the analytical view point? Which idea will make the most interesting high school teaching of mathematics?

If the more general view point is to be introduced, at what stage? But more interesting still is how shall it be done? Would it be wise to lay down Euler's definitions at once and build from them and pass later to geometric applications as a special case? Would it be consistent in the student's mind to have given him at first the traditional definitions and then quite soon tell him they are out of date? Probably some such notion can be put over by showing them that, once the functions of trigonometry are defined and their fundamental properties derived, that much is only a particular case, or merely a graphic representation. What is needed is some detailed statement of how to proceed. Some suggestion as to how to present the periodic character of the functions without the geometrical notion would be really helpful.

If the correspondents of the Mathematics News Letter can make constructive contributions to this problem I shall be very glad to read them, whether they accord with the sentiment set forth above or not. My skin is not so very thin. Neither am I unfriendly to suggestions.

*PLANE GEOMETRY

By ELIZABETH HARRIS
Columbia, Mississippi

Mathematics in general and geometry in particular have a very ancient and honorable history. Of all the sciences geometry's development has been accompanied by the least error. One generation of mathematicians has not been chiefly occupied with the correction of the mistakes of the previous generation but has been free to devote its energies to further development of the sciences.

The virtues of mathematics, precision in definition and logical exactness in reasoning, are especially evident in geometry. Its elements—the point, the line, and the plane—are the most universal of all concepts, and have always meant the same thing to all peoples, and this gives a common and undisputed ground for development. On the contrary, what may seem to be beautiful to one people at one time may not seem to be so at a later date. Even the moral values of acts are differently appraised by different ages and by different races, but the geometrical concepts have in them the elements of the unchangeable and eternal. The shortest distance between two points always has and always will bring to the mind the same concept—the line.

Our knowledge of what the ancients knew of geometry is limited to what may be inferred from inscriptions and tablets. Necessity often serves as the mother of invention and hence the earliest geometry deals with the practical problems, such as the measurement of lands. The Babylonions had formulas for areas of squares, rectangles and right triangles. They asserted that the circumference of a circle was three times its diameter. This

*Read March 8 at Cleveland, Mississippi before the La.-Miss. Branch of the National Council of Teachers of Mathematics.

is also the statement in the Bible in I Kings 7:23. The Egyptians whose civilization goes back beyond record, had a practical use for geometry in the surveying of land, especially in the vicinity of the Nile, which so often floods its banks.

How is geometry used? I walk a certain distance; here is the problem of the measurement of distance. I buy a piece of ground; a drawing must be made to indicate its shape and position for the deed; here is a problem of area. A tank must be constructed to hold so many gallons; here is a question of volume. A tunnel is to be made under a river, a bridge is to be constructed, a ship is to be built; all these are but a few of the many thousands of geometric problems. In war there are endless problems of geometry to solve. The position of a big gun of an airplane, or of a submarine, is found by geometrical means.

Geometry teaches the student to separate the essential from the nonessential. The statements of geometry when they have reached a final form are such that a single word cannot be added or taken away without destroying the meaning.

There is a unity, simplicity and beauty in the development of geometry which appeals to many. Lincoln, whose writings and speeches are especially noteworthy for the above qualities, found joy in middle life in reading the proofs of Euclid.

Geometry is especially fitted as a subject of study for the youthful mind. It demands intelligence, imagination and diligence.

Geometry is founded upon statements about which there is universal agreement. History is supposed to deal with facts, but what passes for history in one country will not be accepted in another and even in the same country opinions differ. Moreover, if the facts themselves are not disputed at least their interpretation is called in question. What is music to the Oriental ear is mere noise to the European. Often that which is considered beautiful in one age is derided in the next. But objects like the monuments of Greek architecture which found their inspiration and proportions in the laws of geometry, have found universal admiration and approval, since the fundamental laws of geometry have been the same for all times and for all peoples.

Smith says: "This characterizes the work of Euclid, a collection of the basic propositions of geometry and chiefly of plane

geometry, arranged in logical sequence, the proof of each depending upon some preceding proposition, definition, or assumption (axiom or postulate). The number of propositions in plane geometry included in 'The Elements' (of Euclid) is not entirely certain, owing to some disagreement in the manuscripts, but it was between one-hundred sixty and one-hundred seventy-five. The efforts at revising Euclid have been generally confined to re-arranging his material, to render more modern his phraseology, and to making a book that is more usable with beginners if not more logical in its presentation of the subject. While there has been an improvement upon Euclid in the art of bookmaking and in minor matters of phraseology and sequence, the educational gain has not been commensurate with the effort put forth.

It is evident that not more than twenty-five per cent of propositions have any genuine applications outside of geometry, and that if we are to attempt any applications at all, these must be sought mainly in the field of pure geometry."

Business today is interested in our educational system, challenging us with these charges: (1) The young man and woman sent from our educational institutions today are not able to cope with life; (2) Given a certain problem they are unable to think clearly and constructively to a definite conclusion; and (3), They are poorly prepared in the fundamentals of education.

The second topic vitally concerns us in the teaching of geometry. Some of the chief demands of the newer methods of geometry teaching include the following points:

1. Problems must be based on reality of interest to the pupil and of social value.
2. Instruction in a new topic should be of the "ladder" type, graded in difficulty from easy to hard or from simpler to more elaborate operations.
3. Time should be spent in computing rather than in copying. Mimeographed or printed practice sheets will reduce the eye-strain and tedium that comes from copying equations and drawing figures. Figures should be lettered differently from those of the book.
4. Pupils should be encouraged to verify each new process by those already learned. Accuracy means checking of results.

5. Topics should be organized from the point of view of the learner.

Initiative is the keystone in the triumphal arch of success in geometry. Iniative may be defined as the capacity to start and carry forward lines of thought and action. "Initiative is the human characteristic in which the will to do and the courage to dare, have their origin and being." It is doing the right thing without being told.

In some new text-books, especially those in geometry, there is found a rather timid use of the history of the subject. I believe more stress should be made of this history than heretofore has been made, because it appeals strongly to students. The interest of a pupil taking geometry who perhaps revels in history might be greatly increased and motivated in this way.

If you can get the pupils to approach the problems from what they consider a common sense viewpoint, you are pretty sure to get their interest; once you have aroused their interest, you will find they will work for hours (if need be) in order to solve the problems.

Too much time is given in the training of teachers to the "how" of teaching and not enough to "what" they are going to teach. The teacher should know what is important and should not be too prompt in skipping important principles.

In geometry the students are allowed to memorize the very lettering of the figures in the book, whereas they should prove their theorems with new letters and so learn the reasoning at the bottom of the proofs. The pupils should reason for themselves instead of the teacher's doing it for them. Too little attention is paid to coroilaries, originals and to important properties of figures that will be useful in other branches of mathematics as well as in physics or mechanics.

Numerical geometry or mensuration has long been a part of the seventh and eighth grades. New demonstrative geometry has two huge initial difficulties, one of ideas and the other of methods. Most pupils begin demonstrative geometry with very vague notions of the definitions of geometric figures and little knowledge of their properties. The elementary notions come from experience. "If you do not know what an angle is I can not tell you" said a wise old Greek. Most pupils do not know what an

angle is and they can not use the idea in questions involving size, variation, comparison or proof. The form of argument of demonstrative geometry is new, concise and formal. It is a difficult tool to use at all, and requires much practice, to be used with facility and accuracy. Hence when a pupil starts applying the new method to a set of ideas that are vague, and when, as a last straw we start him to proving, not the facts that may seem to him to need proof, but those that are quite evident, no wonder there is confusion.

Here again there has been improvement recently. Texts in geometry for the tenth grade are now beginning with a chapter designed to illustrate geometric illustrations, definitions, properties, relation and the need for geometric relations. This if well taught may greatly lessen the initial difficulties. We need a longer introductory course with definite objectives and with enough time on the programme to realize them.

These are some of the ends to which such a course should contribute:

1. Useful information, including that now given in mensuration.
2. The use of the ruler, compasses and protractor as tools for solving practice problems and in discovering and testing geometrical truths.
3. Appreciation of the value of geometry in industry and art.
4. Feeling the need for geometric proofs and making easy ones.

Historically, mathematics was called into existence because of the need for counting and measuring. This fact has given to mathematics a double foundation, namely arithmetic and geometry. It is apparent that we can not make or manufacture the simplest article without giving due attention to its form, its dimensions and the proper relation of its parts. Nature and the manual arts are readily seen to be the two permanent sources of geometry. More especially training in space, intuition and plastic thinking is at the bottom of all forms of applied art. The geometric principles of equality, symmetry, congruence, and similarity are implanted in the very nature of things. The art of

measurement permeates the fabric of modern civilization at every point.

The method of procedure is from the general to the specific case.

In the newer geometries some of the historical problems are those such as Ceva's problem, problems of the nine-point circle or those of Apollonius. Most of those of Apollonius have already been included in geometry.

Some of the aims of geometry are:

1. To develop a concept of space.
2. To train in logical thinking.
3. To train in functional thinking.
4. To appreciate facts.
5. To give an introduction to the study of science and philosophy.
6. To train in exact use of terms.

Much time should be given to original work. A sufficient number of originals and computations should be given to show the pupil the practical application of the proposition.

Pupils should be urged to commit to memory definitions, axioms, propositions and corollaries as they are stated in the text.

Figures should be drawn quickly and accurately, different lettering from that of the text being used. Use only figures and lettering for black-board work after correct form is mastered. Much time is lost in having each student write out the proof in full. Usually no time remains for class criticism or discussion when the proof is written out.

A summary of related propositions and definitions should be made from time to time.

Be sure that the pupil understands the meaning of geometry. It is a new subject to him and the teacher must see that he knows what it takes to learn it.

Spend some time teaching and illustrating the different terms used in geometry. Have the pupils construct, out of wood or cardboard the different figures—angles, triangle, circle, rectangle, parallelogram, etc., and use them in class sometimes instead of drawing figures on the board.

Assign short lessons at first, don't let any of the pupils get

discouraged or get behind. See that every member of the class knows how to read angles and other figures.

In assigning a new lesson, explain it and point out what the pupil is expected to learn, or do in preparation of the lesson. Then see that the pupil does it.

Don't depend on oral work alone in preparation of the lesson, but require the problems to be worked, written out and handed in at least two or three times a week. Grade or correct these papers and explain the corrections in class at least once a week. Don't let the class get careless in preparing these papers. Always call for them when they've been told to prepare them.

Insist upon accurate and neat figures. Each pupil needs a ruler and compasses.

Frequent reviews at first are essential. Drill is very necessary.

As often as you have time—and that depends on the size of the class—have each pupil draw the figure on the board and without reference to a book explain a proposition to the class.

See that the star pupil isn't the only one that gets to explain a proposition. However always recognize the work of superior pupils.

Insist that pupils do original work—especially should the written work not be copied from other pupils' papers.

A prize of an extra grade—or something similar—to the pupils that work a certain problem without help, will stimulate original work.

Give frequent review tests. The teacher should keep in close touch with all her pupils. The best pupils are not the ones that require the most time.

Insist that the pupils do not memorize the proofs of the propositions. In order to discourage it, use different figures with different letters, etc. Let them see that many different figures may fit one proposition just as well as the figure in the book. Be careful that the pupils use figures to illustrate the general rather than the specific case.

About once a week, let the pupils give orally the propositions and corollaries they have had that week.

There are three common methods by which to attack a new problem:

(1) By synthesis.
(2) By analysis.
(3) By the interesection of loci.

Webb's Geometry Test and the Minnick Geometry Tests will be helpful to the teacher in seeing what she has accomplished in her work as a teacher.

We must not confuse the idea of making mathematics interesting with that of making it easy. This latter is a thing that cannot be done and still have the pupils really learn mathematics. The story goes that a prince once asked a Greek mathematician if there were not an easy way to learn geometry. The mathematician replied: "There is no royal road to geometry."

MATHEMATICS FOR SERVICE

By DORA M. FORNO
New Orleans Normal School

Mathematics of the present day differs from that of the past in its greater emphasis upon utility and in the particular applications made of mathematical principles and processes. Society is demanding an education that prepares for living and a changed aspect is noticeable in the socialized nature of the mathematical content of textbooks and other local problem material used in developing principles and processes and in their applications.

In the early elementary grades an effort is made to show how arithmetic helps in solving our every-day problems. Special attention is given to "store" arithmetic, and the "play store" has become a special project whereby the fundamental processes can be rationalized and be made very real. The arithmetic of the children's games leads gradually to making the learning of arithmetic itself a big game wherein each pupil competes against his own past record, or one group competes with another group, or a group aims to come up to a standard set for homogeneous groups.

It requires sympathetic direction to carry on work along these lines but the results are commensurate with the labor expended. The "hit or miss" method of teaching no longer satisfies

the progressive teacher. Progressive teaching and individual instruction are keynotes of present day instruction. The measurement of progress of individuals and of classes by means of graphs does more for vitalizing teaching than any other device. Some of my classes make individual graphs and attach them to the front pages of textbooks and measure the results of a ten-minute drill on certain processes studied. At other times we make only a class graph of two competing teams.

A helpful plan for motivating the study of the fundamental facts and processes is to have supplementary drill material available for the pupils when they finish the regular assigned seat-work. A table should be set apart in some part of the room with interesting material arranged so that it can be easily accessible. Flash cards, games, and timed sheets that may be easily checked by reference to answers on reversd sides furnish abundant material. The training in self-direction and self-reliance which this phase of supplementary work gives is inestimable. The students can be directed to work in teams and drill each other in a quiet orderly way without interfering with the work of another class that is in progress at the same time.

The foundations of the laboratory method of teaching mathematics are laid in these early elementary grades and carried on in the upper grades, when, under proper guidance, the pupils work out their own projects and solve their own problems and do not merely imitate by following blindly a set form of solution. Teachers neglect a big opportunity if they fail to have mathematical laboratory equipment to give concreteness and greater reality to the study of the subject. This equipment, however simple it may be, affords a stimulus to work out problems of construction, observational geometry, etc. The atmosphere of the classroom, and the spirit and attitude of the teachers and pupils have much to do with the success of the work in mathematics in all grades, but it is most important that the basic work in mathematics,—the arithmetic of the elementary schools and first year algebra, should be guarded most carefully that the aims set up in the different grades shall be attained.

Good business practice in teaching is as important as good business practice in other lines of endeavor and should demand taking inventories, keeping careful account of progress, prevent-

ing errors, and building up habits of right thinking and accuracy of statement and form.

A "clinic" for poor students who do not reach the required goals should be organized and a diagnosis made of these failures. Investigators in all lines of work are convinced that prevention is more important than an attempt to salvage wrecks, so it seems that the big task of the teacher is to prevent failures if possible. A folder published by the World Book Co. entitled, "The Smooth Mathematics Course for Direct Rapid, and Efficient Progress" is inspirational. It outlines an articulated course in mathematics for all grades, one through twelve, with complete instructional material and a program of testing. Such a course as outlined should be a panacea for mathematical ills of the elementary and high schools if faithfully followed:

1. The old dangerous curve between informal number work and formal arithmetic is avoided;
2. For junior high school, the rough spot of transition to general mathematics is eliminated;
3. The frequently insurmountable hill of first year algebra is leveled;
4. The difficult spots in beginning geometry or advanced algebra are made easy.

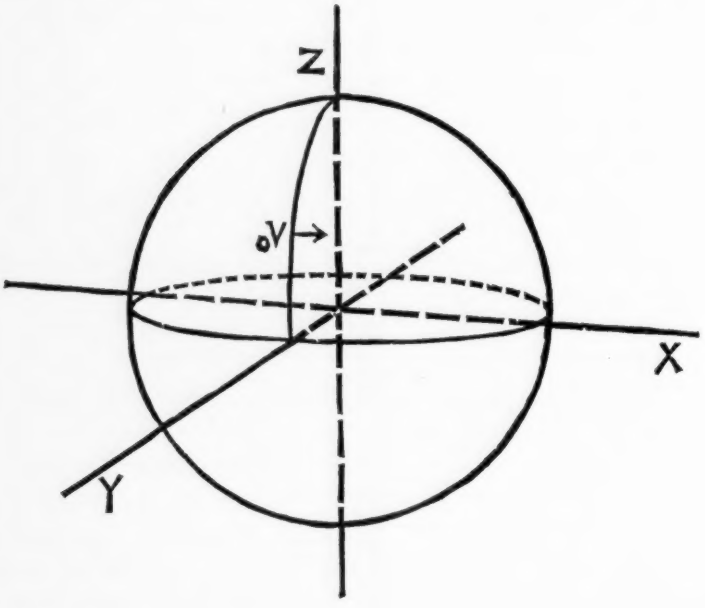
The four points outlined above represent four big problems in elementary and junior high school mathematics. The duplication of subject matter and the lack of articulation between courses causes much loss of time and interest. Much more effective teaching could be done if the problems were more carefully selected and useless problems "scrapped" and the time spent in concentration on problems of greater significance. The power to reason well would be developed more effectively in reasoning about things of interest and of practical value.

ON THE PATH OF A BODY FALLING FREELY WITHIN THE EARTH

By F. A. RICKEY
Louisiana State University

Any attempt to answer the question "How should a hole

be bored through the earth to allow an object to fall freely through it?" can be accurate only to the degree of accuracy of the assumptions which we make concerning the earth's shape and distribution of matter. Let us assume that the earth is a homogeneous sphere, that the object falls in a vacuum, and let us neglect the motion of the earth other than its rotation as we are considering a motion relative only to the interior of the earth. Setting up rectangular axes, the z-axis being the axis of



the earth, the x and y axes lying in the plane of the equator, let the point at which the body is to be released be on the intersection of the surface of the sphere with the yz-plane. The motion of the falling body is then determined by an initial tangential velocity v_0 due to the earth's rotation and by the force of gravity. Since the acceleration of a freely moving body within a homogeneous sphere is known to be directed toward the center and proportional to the distance from the center, we have,

$$d^2r/dt = -k^2r$$

where $k^2 = g/R$ (R being the earth's radius), for $d^2r/dt^2 = -g$ when $r = R$.

We now resolve the acceleration d^2r/dt^2 into its rectangular components, getting

$$d^2x/dt^2 = -k^2x; \quad d^2y/dt^2 = -k^2y; \quad d^2z/dt^2 = -k^2z$$

By use of the integrating factors $2dx/dt$, $2dy/dt$, and $2dz/dt$, respectively, we can readily integrate and obtain,

$$(dx/dt)^2 = -k^2x^2 + c_1; \quad (dy/dt)^2 = -k^2y^2 + c_2; \quad (dz/dt)^2 = -k^2z^2 + c_3$$

Here we observe that $dx/dt = v_0$ when $x = o$, $dy/dt = o$, when $y = y_0$, and $dz/dt = o$ when $z = z_0$, so that the constants of integration can be evaluated and the above equations written

$$dx/dt = k\sqrt{v_0^2/k^2 - x^2}, \quad dy/dt = -k\sqrt{y_0^2 - y^2}; \quad dz/dt = -k\sqrt{z_0^2 - z^2}$$

where the negative signs have been chosen because the motion tends to decrease y and z.

Separating variables and integrating, we have,

$$\sin^{-1}(kx/v_0) = kt + c_4; \quad \cos^{-1}(y/y_0) = kt + c_5; \quad \cos^{-1}(z/z_0) = kt + c_6$$

When $t = o$, $x = o$, $y = y_0$, and $z = z_0$, hence the three constants of integration are each equal to zero and we write the equations:

$$\sin kt = kx/v_0; \quad \cos kt = y/y_0; \quad \cos kt = z/z_0 \qquad (1)$$

or

$$x = v_0/k \sin kt, \quad y = y_0 \cos kt; \quad z = z_0 \cos kt \qquad (1')$$

the parametric equations of the curve followed by the falling body within the earth's sphere.

To determine the actual path through the earth, we must offset the rotation of the earth by a corresponding rotation of axes about the z—axis. If w is the angular velocity of the earth's rotation, wt is the angle of rotation of axes, and since z is unchanged by the rotation of the x and y axes we use the usual formulas of negative rotation on x and y, the only difference being that here the angle of rotation is a function of the time. Equations (1^1) then reduce to:

$$\left.\begin{aligned} x' &= v_0/k \sin kt \cos wt - y_0 \cos kt \sin wt \\ y' &= y_0 \cos kt \cos wt + v_0/k \sin kt \sin wt \\ z' &= z_0 \cos kt \end{aligned}\right\} \qquad (2)$$

the desired equations. Attention is called to the fact that equations (1') represent the actual path of motion of the object, while equations (2) represent the route of motion through the earth.

If $z = o$, the first two equations of (1^1) and of (2) describe the motion of an object falling freely in the earth from a point

on the equator. Squaring and adding the first two equations of (1), we have as the path of the object the ellipse (y_0 becoming R)

$$k^2x^2/v_0^2+y^2/R^2=1$$

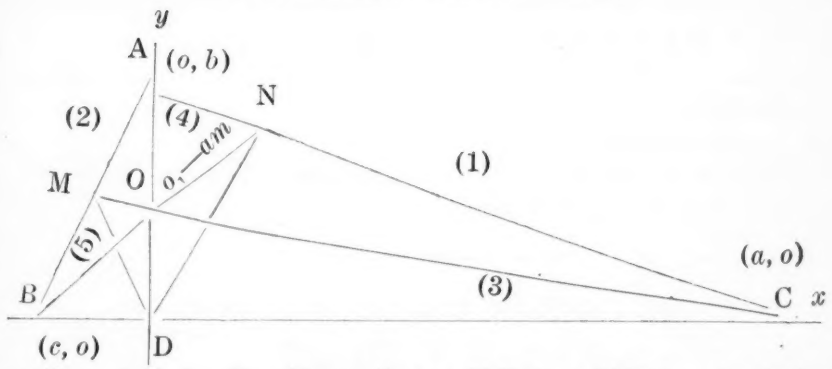
from which we see that the nearest distance the object comes to the center of the earth (the minor semi- axis) is v_0/k, approximately 236 miles. The time of passage through the earth can be determined by setting $y=-R$ (also $y_0=R$) in the second equation of (1), getting $\cos kt=-1$, and hence $t=\pi/k$ or about 42.5 minutes. It is interesting to note that while the object moves in an ellipse, its true path through the earth in an arc which is convex toward the center of the earth.

Our conclusions in brief are: that if an object could fall without resistance through the earth, it would reach the surface on the other side in the time $t=\pi/k=\pi\sqrt{R/g}$, =42.5 min. (approximately); the only point at which it could be dropped so that its path would be a straight line would be one of the poles of the earth; at all other points of release its path would be a plane curve, an ellipse, lying in the plane determined by the direction of the initial velocity and by the earth's center; its path **through the earth** would be in general a skew curve concave toward the earth's surface, being a plane curve only when the point of release is on the equator; an object dropped into the earth at a given point would reach the surface at a point, not on the opposite end of the diameter from the given point, but differing in longitude from such a location by an angle equal to the angle turned through by the earth during the time of passage, $t=\pi/k$. Thus an object released at 0° longitude and 30° *n.* latidude would emerge some 42.5 minutes later at 30° *s.* latitude and approximately 169.4° longitude east of 0°.

ANALYTICAL SOLUTION OF AN INTERESTING PROBLEM

By JAS. P. COLE
L. P. I., Ruston, La.

Given any $\triangle$ as A B C, with A D as one altitude.
Draw C M any line intersecting A D in O. Draw B O inter-



secting A C in N. Show that ∟N D A=∟M D A. Analytical proof:

Let coordinates of C be (a, o); of A, (o, b); of B, (c, o), D being the origin of rectangular axes. Then

equation of line A C is $y=(x-a)(-b/a)$ - - - - (1)

while that of line A B is $y=(x-c)(-b/c)$ - - - (2)

Let slope of C M $=m$

Then equation of C M is $y=m(x-a)$ - - - - - - - (3)

and of A D is $x=0$ - - - - - - - - - - - - (4)

We simultate equations (3) and (4).

Coordinates of O are $(o,-am)$

Equation of BO is $y=(x-c)(am/c)$ - - - - - - - - (5)

Simultating (3) and (2),

$$\left.\begin{aligned} y&=\frac{bcm-bam}{cm+b}\\ x&=\frac{acm+bc}{cm+b}\end{aligned}\right\}\text{ Coordinates of M}$$

Simultating (1) and (5)

$$\left.\begin{aligned} x&=\frac{acb+a^2cm}{a^2m+cb}\\ y&=\frac{a^2mb-abcm}{a^2m+cb}\end{aligned}\right\}\text{ Coordinates of N}$$

Slope of M D is $\dfrac{bcm-bam}{acm+bc}$ - - - (6)

Slope of N D is $\dfrac{a^2mb-abcm}{abc+a^2cm}$ - - - (7)

If $\angle s$ N D A and M D A are equal
$\angle s$ C D N and B D N are equal.

Then the slope of N D = —slope M D
but (6) = —(7)

Since m is any slope, C M can intersect A D at any point and the $\angle s$ N D A and M D A would be equal. q. e. d.

ON CERTAIN USEFUL FUNCTIONS

By H. L. SMITH
Louisiana State University

In this paper we shall consider certain useful functions which take on at most three different values.

1. **The Kronecker function** $d(x)$. Let

$$d(x)=0, \quad x<0,\ x>0$$
$$=1, \quad x=0.$$

This function has the following obvious properties:

(1) $d(-x)=d(x)$

(2) $d(x)^2=d(x)$

(3) $d(x-y)=0, \quad x<y,\ x>y,$
$=1, \quad x=y.$

(4) If $a_1, \ldots a_n$ are n numbers, then
$d(a-a_1)+d(a-a_2)+\ldots+d(a-a_n)$
represents the number of numbers in the set $a_1, \ldots, a_n$ which are equal to a.

2. **The function** $s(x)$. Let

$$s(x)=1, \quad x>0,\ x=0$$
$$=-1, \quad x<0$$

Then

(1) $s(x)^2=1$

(2) $|x|=x\,s(x)$

(3) $s(x)=1+2d(x)-2d(x+|x|)$

(4) $d(x)=(1/2)\,[s(x)+s(-x)]$.

This is a very useful function. For example by its aid we can express a solution of the system of equations

(See Webber and Smith, Junior College Mathematics, vol. II, preliminary mimeographed edition, p. 195).

$x \cos y = a, \quad x \sin y = b$

as follows:

$x = \sqrt{a^2+b^2}, \; y = 2n\pi + s(b) \arccos (a/\sqrt{a^2+b^2})$.

3. **The function** $\operatorname{sgn} x$. Let

$$\begin{aligned} \operatorname{sgn} x &= 1, \quad x>0 \\ &= 0, \quad x=0 \\ &= -1, \quad x<0. \end{aligned}$$

Then:

(1) $\operatorname{sgn} x = (2/\pi) \lim_n \arctan nx$

(2) $\operatorname{sgn}(-x) = -\operatorname{sgn} x$

(3) $\operatorname{sgn} x = 1 + d(x) - 2d(x+|x|)$

(4) $d(x) = 1 - \operatorname{sgn}^2 x$

(5) $s(x) = 1 + \operatorname{sgn} x - \operatorname{sgn}^2 x$

(6) $|x| = x \operatorname{sgn} x$

(7) $\operatorname{sgn} x = s(x) - d(x)$

(8) $\operatorname{sgn} x = \frac{1}{2}[s(x) - s(-x)]$.

4. **The function** $u_0(x)$. Let

$$\begin{aligned} u_0(x) &= 1, \quad x>0 \\ &= \tfrac{1}{2}, \quad x=0 \\ &= 0, \quad x<0. \end{aligned}$$

Then

(1) $u_0(x) = (\frac{1}{2}) + (\frac{1}{2}) \operatorname{sgn} x$

(2) $\operatorname{sgn} x = 2u_0(x) - 1$.

(3) $u_0(x) = 1 + (\frac{1}{2}) d(x) - d(x+|x|)$

(4) $d(x) = 4u_0(x)[1 - u_0(x)]$.

(5) $s(x) = -1 + 6u_0(x) - 4u_0(x)^2$.

(6) $u_0(x) = (\frac{1}{2}) + (\frac{1}{4})[s(x) - s(-x)]$.

5. **The function** $u_1(x)$. Let

$$\begin{aligned} u_1(x) &= 1, \quad 0<x<1, \\ &= \tfrac{1}{2}, \quad x=0, \; x=1, \\ &= 0, \quad x<0, \; x>1. \end{aligned}$$

Then

(1) $u_1(x) = u_0(x)\, u_0(1-x) = u_0(x-x^2)$

(2) $u_1 \dfrac{(x-a)}{(b-a)} = 1, \quad a<x<b,$

$$\begin{aligned} &= \tfrac{1}{2}, \quad x=a, \; x=b, \\ &= 0, \quad x<a, \; x>b. \end{aligned}$$

$$(3)\quad u_1\frac{(x-a)}{(b-a)}=u_0[(x-a)(b-x)]=(1/2)+(1/2)\operatorname{sgn}\frac{}{(x-b)(x-b)}$$

This function is useful in the construction of functions which are defined by different formulas in different parts of their range of definition. Thus we may write parametric equations of the triangle whose vertices are $(x_0y_0),(x_1y_1),(x_2y_2)$ thus

$$x=[x_0+(x_1-x_0)t]u_1(t)+[x_1+(x_2-x_1)(t-1)]u_1(t-1)$$
$$+[x_2+(x_0-x_2)(t-2)]u_1(t-2),$$
$$y=[y_0+(y_1-y_0)t]u_1(t)+[y_1+(y_2-y_1)(t-1)]u_1(t-1)$$
$$+[y_2+(y_0-y_2)(t-2)]u_1(t-2).$$

VARIOUS COMMENTS ON THE NEWS LETTER

Naperville, Illinois, October 1, 1929.

Prof. S. T. Sanders, Editor
Dear Sir:

Some weeks ago there came to my desk a copy of your Mathematics News Letter.

In it I found some very valuable material. So I am writing asking if I can get all the back numbers. I see the pamphlet I have is No. 4 of Vol. 3. If so please inform me what all the preceding will cost me. I think I would like to have the file complete to date and then subscribe for future issues. I want my class in methods to have access to some of the fine things I feel sure they will find in them (judging from the present No.)

Hoping to have an early reply I am.

Yours respectfully,
M. W. COULTRAP.

Winter Park, Florida, January 28, 1929.

Mathematics News Letter
Baton Rouge, La.
Dear Professor Sanders:

A belated copy of the NEWS LETTER reached me and I do not want to lose any time in sending in my small subscription for same for the current year.

A paper of this kind is what is needed and will I am sure produce much good in the field of Mathematics.

Anxiously awaiting the next copy and wishing you every success, I am. Very truly,

EDWARD F. WEINBERG.

Rock Hill, S. C., February 9, 1930.

Prof. S. T. Sanders,
Editor-in-Chief of the Mathematics News Letter,
Baton Rouge, La.
Dear Prof. Sanders:

This is such a delayed acknowledgment of your letter requesting an article for the News Letter, that it must be an apology, too.

The News Letter is an interesting journal and is read with real profit each month. I would have felt honored to be privileged to contribute to it. However, I did not have anything of value to send to you.

Enclosed is a check for my subscription. Yours sincerely,

HORTENSE ROGERS.

Peru, Nebraska, April 15, 1929.

Mr. S. T. SANDERS,
Baton Rouge, La.
Dear Sir:

If you have on hand any sample copies of the Mathematics News Letter, and if you have no further use for them, I would appreciate having about fifteen copies for use in our students honorary mathematics club. Yours very truly,

ARTHUR L. HILL.

Shreveport, La., January 17, 1930.

Dear President—

I believe you will be interested in reading a recent letter of Dr. H. E. Slaught, a copy of which I enclose. This letter gives this mathematics leader's appraisal of the cooperative efforts of the Louisiana-Mississippi Sections of the Mathematical Association of America and the National Council of Teachers of Mathematics, and also his estimate of the value of the MATHEMATICS NEWS LETTER, which is the joint organ of these two organizations.

A strong effort is being made to place this important work on a sound financial basis. While the number of subscriptions to the NEWS LETTER is increasing, and several mathematics teachers are making individual contributions, some financial aid is needed from the colleges and universities of the two states.

I mention only one of the several purposes of this enterprise which alone, I think, justifies the financial support of the institutions of high learning in this region. This purpose is, a better preparation of high school graduates for college mathematics. The need in this respect is only too well known to all college administrative officers and teachers of mathematics. Already there are evidence that some high school principals and teachers are becoming more determined to send to the colleges a better product, and college instructors are showing a stronger desire to help them. If the plans, now well started, can continue we have good reason to expect a steady improvements in the present deplorable condition.

If the colleges and universities in the two states will each contribute from ten to fifty dollars a year to the Louisiana-Mississippi Section of the M. A. of A. the continuance and expansion of this work will be assured. Your institution has made some contribution in the past years. May we count on a contribution in the near future for the work of the current year?

Sincerely yours,

JNO. A. HARDIN, Chairman.

Blue Mountain, Miss.

Prof. S. T. Sanders,
Baton Rouge, La.
Dear Prof. Sanders:

I am sending check for two subscriptions to the Mathematics News Letter. Send one copy to Mr. Lacy Hodges, Blue Mountain, Miss., beginning with January, the other to me. I have received my January copy.

The News Letter is good. I enjoy it. With best wishes, I am,

Your friend,

MABEL HUTCHINS.